

Generalized Ricci dark energy in Horava-Lifshitz gravity

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In this letter, we have considered generalized Ricci dark energy in the Horava-Lifshitz gravity. We have reconstructed the Hubble's parameter in terms of fractional densities. We have viewed the equation of state parameter in this situation. Also, we have examined the behavior of deceleration parameter and investigated the nature of the statefinder diagnostics. The equation of state parameter has exhibited quintessence-like behavior and from the plot of the deceleration parameter we have observed an ever accelerating universe.

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The accelerating cosmic expansion of the present universe has been strongly confirmed by the cosmic microwave background radiation (CMBR) [1] and Sloan Digital Sky Survey (SDSS) [2]. An exotic form of negative pressure matter called dark energy (DE) is used to explain this acceleration. Extensive reviews on DE are available in [3], [4], [5] and [6]. The most powerful quantity of DE is its equation-of-state (EoS) effectively defined as $w_{DE} = p_{DE}/\rho_{DE}$, where p_{DE} and ρ_{DE} are the pressure and energy density respectively. All DE models can be classified by the behaviors of equations of state as following [6]:

- Cosmological constant: its EoS is exactly equal to -1 , that is $w_{DE} = -1$.
- Quintessence: its EoS remains above the cosmological constant boundary, that is $w_{DE} \geq -1$.
- Phantom: its EoS lies below the cosmological constant boundary, that is $w_{DE} \leq -1$.
- Quintom: its EoS is able to evolve across the cosmological constant boundary.

An approach to the problem of DE arises from holographic principle that states that the number of degrees of freedom related directly to entropy scales with the enclosing area of the system [7]. Works on *holographic dark energy* (HDE) include the references [8], [9], [10] and [11]. There are many papers about holographic reconstruction of different dark energy models. References in this regard are given in [12]. Inspired by the HDE models, Gao et al [13] proposed a DE model dubbed as "Ricci dark energy" (RDE), where the energy density of the universe is proportional to the Ricci scalar $R = -6 \left(\dot{H} + 2H^2 + \frac{k}{a^2} \right)$, where dot denotes a derivative with respect to time t and k is the spatial curvature. The energy density of RDE is [13]

$$\rho_R = 3c^2 \left(\dot{H} + 2H^2 + \frac{k}{a^2} \right) \quad (1)$$

where c is a dimensionless parameter which will determine the evolution behavior of RDE. When $c^2 < 1/2$, the RDE will exhibit a quintomlike behavior. With the proposed form of energy density, the energy density can be solved from the Friedmann equation as

$$\rho_R = \frac{\alpha}{2-\alpha} \Omega_{m0} e^{-3 \ln a} + f_0 e^{-(4-\frac{\alpha}{2}) \ln a} \quad (2)$$

where, $\alpha = \frac{8\pi c^2}{3}$. Thus the RDE has one part which evolves like nonrelativistic matter ($\sim e^{-3 \ln a}$) and another part which is slowly increasing with decreasing redshift [13]. Subsequently, the pressure of RDE can be obtained as [13]

$$p_R = - \left(\frac{2}{3\alpha} - \frac{1}{3} \right) f_0 e^{-(4-\frac{\alpha}{2}) \ln a} \quad (3)$$

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In the studies subsequent to the reference [13], various aspects of RDE have been investigated. In the said reference [13], it was shown that this model can avoid the causality problem and naturally solve the coincidence problem of dark energy. Xu et al [14] revisited the RDE model and used cosmic observations to constrain the model parameters. Feng and Li [15] investigated the viscous RDE model by assuming that there is bulk viscosity in the linear barotropic fluid and the RDE. Feng [16] reconstructed $f(R)$ theory from RDE. A correspondence between various dark energy candidates and RDE was studied by Chattopadhyay and Debnath [17]. A generalized model has been designed by Xu et al [18] to include HDE and RDE by introducing a new parameter which balances holographic and Ricci dark energy model. Reference [18] considered a generalized versions of holographic and Ricci dark energy and taken the energy densities for generalized holographic dark energy (GHDE) and generalized Ricci dark energy (GRDE) as

$$\rho_{GH} = 3c^2 M_{pl}^2 f\left(\frac{R}{H^2}\right) H^2 \quad (4)$$

$$\rho_{GR} = 3c^2 M_{pl}^2 g\left(\frac{H^2}{R}\right) R \quad (5)$$

where $f(x)$ and $g(y)$ are positive defined functions of the dimensionless variables $x = R/H^2$ and $y = H^2/R$ respectively. Holographic and Ricci dark energy models are recovered when the function $f(x) = g(y) \equiv 1$. Also, when the function $f(x) = x$ and $g(y) = y$, the holographic and Ricci dark energy exchange each other. The functions can be written as [18]

$$f\left(\frac{R}{H^2}\right) = 1 - \epsilon \left(1 - \frac{R}{H^2}\right) \quad (6)$$

$$g\left(\frac{H^2}{R}\right) = 1 - \eta \left(1 - \frac{H^2}{R}\right) \quad (7)$$

where ϵ and η are parameters. When $\epsilon = 0 (\eta = 1)$ or $\epsilon = 1 (\eta = 0)$, the generalized energy density becomes the holographic (Ricci) and Ricci (holographic) dark energy density respectively. If $\epsilon = 1 - \eta$, then GHDE and GRDE are equivalent. Endeavor of the present work is to discuss the GRDE in Horava-Lifshitz (HL) gravity [19], [20], [21]. The basic idea of the HL approach is to modify the UV behavior of the general theory so that the theory is perturbatively renormalizable [22]. In this theory the local Lorentz invariance is abandoned, but it is restored as an approximate symmetry at low energies [23]. Review on HL gravity is available in the reference [21].

A. An overview of Horava-Lifshitz gravity

We briefly review the scenario where the cosmological evolution is governed by HL gravity. The dynamical variables are the lapse and shift functions, N and N_i respectively, and the spatial metric g_{ij} . In terms of these fields the full metric is written as [24], [25]

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \quad (8)$$

where indices are raised and lowered using g_{ij} . The scaling transformation of the coordinates reads: $t \rightarrow l^3 t$ and $x^i \rightarrow l x^i$.

The action of the HL gravity is given by [24], [25]

$$\begin{aligned} I &= dt \int d^3x (\mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_m) \\ \mathcal{L}_0 &= \sqrt{g} N \left[\frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda K^2) + \frac{\kappa^2 \mu^2 (\Lambda R - 3\Lambda^2)}{8(1-3\lambda)} \right] \\ \mathcal{L}_1 &= \sqrt{g} N \left[\frac{\kappa^2 \mu^2 (1-4\lambda)}{32(1-3\lambda)} R^2 - \frac{\kappa^2}{2\omega^4} (C_{ij} - \frac{\mu\omega^2}{2} R_{ij})(C^{ij} - \frac{\mu\omega^2}{2} R^{ij}) \right] \end{aligned} \quad (9)$$

where, κ^2 , λ , μ , ω and Λ are constant parameters, and C_{ij} is Cotton tensor (conserved and traceless, vanishing for conformally flat metrics). The first two terms in \mathcal{L}_0 are the kinetic terms, others in $(\mathcal{L}_0 + \mathcal{L}_1)$ give the potential of the theory in the so-called “detailed-balance” form, and \mathcal{L}_m stands for the Lagrangian of other matter field. Comparing the action to that of the general relativity, one can see that the speed of light and the *cosmological* Newtons constant are [20], [24], [29]

$$c = \frac{\kappa^2 \mu}{4} \sqrt{\frac{\Lambda}{1 - 3\lambda}}, \quad G_c = \frac{\kappa^2 c}{16\pi(3\lambda - 1)} \quad (10)$$

It may be noted that when $\lambda = 1$, \mathcal{L}_0 reduces to the usual Lagrangian of Einsteins general relativity. Thus, when $\lambda = 1$, the general relativity is approximately recovered at large distances.

We are considering the existence of both dark energy and dark matter. Using the identifications in (10) one can rewrite the field equations as [29]

$$H^2 + \frac{k}{a^2} = \frac{8\pi G_c}{3} \rho + \frac{k^2}{2\Lambda a^4} + \frac{\Lambda}{2} \quad (11)$$

and

$$\dot{H} + \frac{3}{2}H^2 + \frac{k}{2a^2} = -4\pi G_c p - \frac{k^2}{\Lambda a^4} + \frac{3\Lambda}{4} \quad (12)$$

where, we have defined the Hubble parameter as $H = \frac{\dot{a}}{a}$, Λ is the cosmological constant. The above equations (11) and (12) are derived in reference [29]. The term proportional to a^{-4} is the usual “dark radiation term”, present in HL cosmology. Also, $\rho = \rho_X + \rho_m$ and $p = p_X + p_m$. Here, X denotes the dark energy component and m denotes the dark matter component. As there is no interaction, we have the conservation equations

$$\dot{\rho}_X + 3H(\rho_X + p_X) = 0; \quad \dot{\rho}_m + 3H(\rho_m + p_m) = 0 \quad (13)$$

B. GRDE in Horava-Lifshitz gravity

From equations (5) and (6) we get the energy density of GRDE (we assume $M_{pl}^2 = 1$). Thus,

$$\rho_X = 3c^2 \left[-6 \left(2H^2 + \dot{H} + \frac{k}{a^2} \right) (1 - \eta) + H^2 \eta \right] \quad (14)$$

From the conservation equation (13)

$$\rho_m = \rho_{m0} a^{-3(1+w_m)} \quad (15)$$

Consequently,

$$H^2 = \frac{8\pi G_c}{3} \left[3c^2 \left\{ -6 \left(2H^2 + \dot{H} + \frac{k}{a^2} \right) (1 - \eta) + H^2 \eta \right\} + \rho_{m0} a^{-3(1+w_m)} \right] + \frac{k^2}{2\Lambda a^4} + \frac{\Lambda}{2} - \frac{k}{a^2} \quad (16)$$

In (16), H^2 appears both in the left and the right hand side and \dot{H} appears in the right hand side. Solving the ordinary differential equation (16) we can express H^2 as a function of the scale factor a as

$$H^2 = \frac{1}{2} \left[\frac{\Lambda}{1 + c^2(96 - 95\eta)} + \frac{1}{a^4} \left\{ \frac{k^2}{\Lambda(1 + c^2\eta)} + \frac{2a^2 k(1 - 48c^2(-1 + \eta))}{-1 + c^2(48 - 47\eta)} + 2a^{\frac{1+c^2\eta}{24c^2(-1+\eta)}} C_1 \right\} \right] - \frac{\rho_{m0} a^{-3(1+w_m)}}{3(-1 + c^2(-96 + 95\eta))} \quad (17)$$

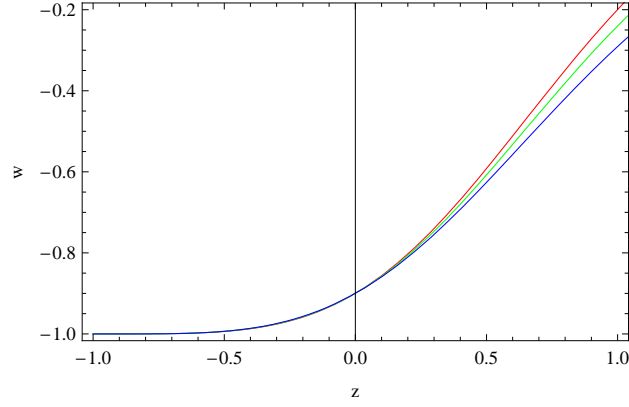


Fig.1

Fig. 1 shows the variation of the equation of state parameter w against z in for GRDE in the Horava-Lifshitz gravity.

The red, green and blue lines correspond to $c^2 = 0.2, 0.5$ and -0.8 respectively. Also, we have taken

$$\Omega_{mo} = 0.27, \quad \Omega_{k0} = 5 \times 10^{-5}, \quad \Omega_{\Lambda 0} = 1 \times 10^{-6}.$$

Dividing both sides of the above Friedman equation by $3H_0^2$ we get

$$\mathcal{H}^2 = \frac{(1 - 48c^2(-1 + \eta))\Omega_{k0}}{a^2(-1 + c^2(-48 + 47\eta))} + \frac{\Omega_{k0}^2}{2a^4(1 + c^2\eta)\Omega_{\Lambda 0}} + \frac{\Omega_{\Lambda 0}}{2(1 + c^2(96 - 95\eta))} + \frac{a^{-3(1+w_m)}\Omega_{m0}}{3(-1 + c^2(-96 + 95\eta))} + a^{-4 + \frac{1+c^2\eta}{24c^2(-1+\eta)}} f_0 \quad (18)$$

where, the relative densities are

$$\Omega_{\Lambda 0} = \frac{\Lambda}{3H_0^2}; \quad \Omega_{k0} = \frac{k}{3H_0^2}; \quad \Omega_{m0} = \frac{\rho_m}{3H_0^2} \quad (19)$$

and $f_0 = \frac{C_1}{3H_0^2}$ is the integration constant. Using the above expression of \mathcal{H}^2 in (14) after dividing (14) by $3H_0^2$ we get

$$\Omega_{X0} = 3c^2 \left[\frac{a^{\frac{1}{24} \left(-95 + \frac{1+c^2}{c^2(-1+\eta)} \right)} f_0 (1+9c^2\eta)}{8c^2} + \frac{(1-54c^2(-1+\eta))\eta\Omega_{k0}}{a^2(-1+c^2(-48+47\eta))} + \frac{\eta\Omega_{k0}^2}{2a^4\Omega_{\Lambda 0}(1+c^2\eta)} - \frac{a^{-3(1+w_m)}(-12+13\eta)(3a^{3(1+w_m)}\Omega_{\Lambda 0}-2\Omega_{m0})}{6(-1+c^2(-96+95\eta))} \right] \quad (20)$$

Using (18) and (20) in the conservation equation and considering w_0 as the present value of the EoS parameter i.e. $w_0 = p_X/\rho_X$ ($a = 1$), the expression for f_0 can be determined as

$$f_0 = -\frac{192c^4(-A_1 + 3A_2(1 + 3w_0))(1 - \eta)}{(1 + 9c^2\eta)(1 + 24c^2(1 - 33w_0) - c^2\eta(23 - 72w_0))} \quad (21)$$

where,

$$A_1 = \frac{2(-1 + 54c^2(-1 + \eta))\eta\Omega_{k0}}{-1 + c^2(-48 + 47\eta)} - \frac{2\eta\Omega_{k0}^2}{(1 + c^2\eta)\Omega_{\Lambda 0}} \quad (22)$$

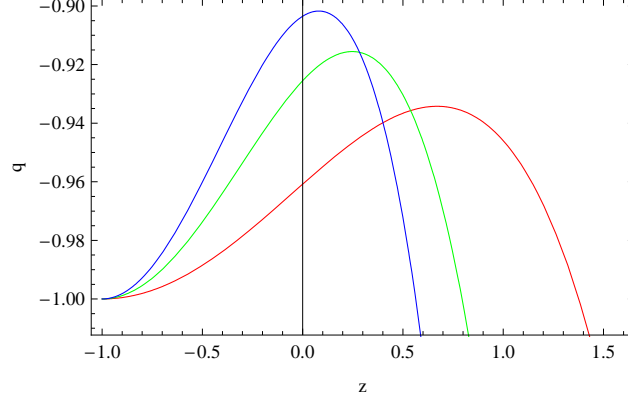


Fig.2

Fig. 2 shows the variation of the deceleration parameter q against z in for GRDE in the Horava-Lifshitz gravity. The red, green and blue lines correspond to $c^2 = 0.2, 0.5$ and -0.8 respectively. Also, we have taken $\Omega_{mo} = 0.27, \Omega_{k0} = 5 \times 10^{-5}, \Omega_{\Lambda 0} = 1 \times 10^{-6}$.

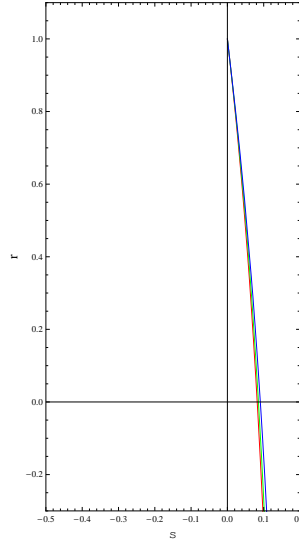


Fig.3

Fig. 3 shows $r - s$ trajectory for GRDE in the Horava-Lifshitz gravity. The red, green and blue lines correspond to $c^2 = 0.2, 0.5$ and -0.8 respectively.

$$A_2 = \frac{(-1 + 54c^2(-1 + \eta))\eta\Omega_{k0}}{-1 + c^2(-48 + 47\eta)} - \frac{\eta\Omega_{k0}^2}{2(1 + c^2\eta)\Omega_{\Lambda 0}} - \frac{a^{-3(1+w_m)}(12 - 13\eta)(3a^{3(1+w_m)}\Omega_{\Lambda 0} - 2\Omega_{m0})}{6(-1 + c^2(-96 + 95\eta))} \quad (23)$$

Now we consider the deceleration parameter $q =$ for the same model. The deceleration parameter is given by [26]

$$q = -1 - \frac{\dot{H}}{H^2} = -1 - \frac{a}{2\mathcal{H}^2} \frac{d\mathcal{H}^2}{da} \quad (24)$$

Using the results obtained above we get the form of q as

$$q = -1 - \frac{a^{-2+\frac{1+c^2\eta}{48c^2(-1+\eta)}} f_0 \left(-4 + \frac{1+c^2\eta}{24c^2(-1+\eta)} \right) - \frac{(1-48c^2(-1+\eta))\Omega_{k0}}{a^2(-1+c^2(-48+47\eta))} - \frac{\Omega_{k0}^2}{a^4\Omega_{\Lambda 0}(1+c^2\eta)}}{a^{-5+\frac{1+c^2\eta}{24c^2(-1+\eta)}} f_0 + \frac{(1-48c^2(-1+\eta))\Omega_{k0}}{a^2(-1+c^2(-48+47\eta))} + \frac{\Omega_{k0}^2}{2a^4\Omega_{\Lambda 0}(1+c^2\eta)} + \frac{\Omega_{\Lambda 0}}{2(1+c^2(96-95\eta))} + \frac{a^{-3(1+w_m)}\Omega_{m0}}{3(-1+c^2(-96+95\eta))}} \quad (25)$$

Now we discuss the statefinder diagnostic pair i.e., $\{r, s\}$ parameters that are of the following form [27]

$$r = \frac{\ddot{a}}{a\mathcal{H}^3} = 1 + 3\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{\ddot{\mathcal{H}}}{\mathcal{H}^3} \quad (26)$$

and

$$s = \frac{r-1}{3\left(q-\frac{1}{2}\right)} = -\frac{3\mathcal{H}\dot{\mathcal{H}} + \ddot{\mathcal{H}}}{3\mathcal{H}(2\dot{\mathcal{H}} + 3\mathcal{H}^2)} \quad (27)$$

The $\{r, s\}$ pair is now constructed based on \mathcal{H} expressed earlier. The $r-s$ plane would be constructed and an $r-s$ trajectory would be created and would be discussed in the next subsection.

C. Discussion

In this work we have considered generalized Ricci dark energy in the Horava-Lifshitz gravity. We have reconstructed the Hubble's parameter in terms of fractional densities. We have viewed the equation of state parameter in this situation. In figure 1 we have plotted the equation of state parameter w for generalized Ricci dark energy taking $c^2 = 0.2, 0.5$ and -0.8 . We have taken $\eta = \frac{1}{13}$ and $w_0 = -0.9$. For different values of c^2 we plot w . For the current model we find that the equation of state parameter stays above -1 for all values of c^2 . This indicates quintessence like behavior. The deceleration parameter expressed above is presented in figure 2, which shows that q is staying in the negative side for all redshifts in the case of GRDE considered in Horava-Lifshitz gravity. This indicates an ever accelerating universe. The $r-s$ trajectory is presented in figure 3. Here we find that for all values of c^2 the $r-s$ trajectory is confined within the first and fourth quadrant of the $r-s$ plane. The trajectory begins from $\{r=1, s=0\}$ and then $r \rightarrow -\infty$ for finite s . Thus we see that the trajectory passes through and not go beyond the Λ CDM phase of the universe when we consider generalized Ricci dark energy in Horava-Lifshitz gravity.

The present study deviates from the earlier studies in the following aspects: Contrary to earlier works [18], [15] done on Einstein gravity the present study is made in Horava-Lifshitz gravity, which is a modified gravity theory. In [16], Ricci dark energy was considered in $f(R)$ gravity. In the present paper we have considered generalized Ricci dark energy in Horava-Lifshitz gravity. In reference [30], Ricci dark energy was considered on chameleon field with the choice of scale factor in the form of emergent universe. However, in the present paper we have not considered any particular choice of scale factor. A study of statefinder diagnostics of Ricci dark energy was done by [31], where it was found that the $\{r-s\}$ trajectory is a vertical segment, i.e. s is a constant during the evolution of the universe for a particular choice of c^2 . However, when we consider generalized Ricci dark energy in Horava-Lifshitz gravity, we find that the trajectory begins from $\{r=1, s=0\}$ and then $r \rightarrow -\infty$. In the work by [28], c^2 was taken as 0.3, 0.4, 0.5, and 0.6 and it was observed that for $c^2 < 1/2$ the equation of state parameter $w < -1$, i.e. behaves like quintom and for $c^2 > 1/2$ the equation of state parameter $w > -1$, i.e. behaves like a quintessence. However, the present study shows that when generalized Ricci dark energy is considered in Horava-Lifshitz gravity, the equation of state parameter always exhibits quintessence-like behavior irrespective of $c^2 >, =, < 1/2$.

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